

# Dependence between Croatian and European stock markets – A copula GARCH approach\*

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## Abstract

*The objective of this paper is to analyze dependence structure between the returns of Croatian and five European stock markets (Austrian, French, German, Italian, and the U.K.'s). We propose a copula GARCH approach, where the return series are modeled as univariate GARCH processes and the dependence structure between the return series is defined by a copula function. Four different copulas are fitted – a constant and conditional normal and symmetric Joe-Clayton (SJC) copulas – and estimated by a semi-parametric method. We found that the time-varying normal copula yields the best fit for CROBEX-CAC40, CROBEX-DAX, and CROBEX-FTSE-MIB stock indices pairs, while the time-varying SJC copula is the best fit for CROBEX-ATX and CROBEX-FTSE100. Further, we found that the probability of simultaneous extreme positive and negative returns in Croatian and other European stock markets can increase to 0.77 during turbulent times. The lower and upper tail dependence dynamics between Croatian and other European stock markets is similar in pattern, differing only in scale. The basic conclusion of the research is that the dependence between the stock markets of Croatia and five major European stock markets is dynamic and can be properly captured by either a dynamic normal or symmetrized Joe-Clayton copula GARCH models.*

**Key words:** stock market, dependence, copula GARCH

**JEL classification:** G11, G15

## 1. Introduction

In the last few years European stock markets have experienced several periods of distress. The shocks in financial market spread quickly between markets and across countries, causing severe cost to market participant. It is also for this reason that measuring the dependence in international financial markets during

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extreme fluctuations has received considerable attention in recent years. There is an increasing amount of evidence in existent literature that stock returns are more interdependent when returns are extremely negative (lower tail dependence) than when they are extremely positive (upper tail dependence), a phenomenon which cannot be captured by simple correlation (Longin and Solnik, 2001; Ang and Chen, 2002; Capiello et al., 2003; Hong et al., 2007; Boero et al. 2009, Dajcman et al. 2012). Simple (Pearson's) correlation measure is a good tool to measure the dependence only in the multivariate normal distribution setting<sup>2</sup>.

The structure of dependence between stock markets has important implications for financial market participants. As Hong et al. (2007) show, incorporating the asymmetry of correlations into portfolio decisions can add substantial economic value to investors. The dependence structure is also important for supervisory authorities and central bank, because of implications the dependence between the financial markets and the institutions of systemic importance have on the stability of financial markets (Clare and Lekkos, 2000; Berben and Jansen, 2005).

The main aim of this study is to exploit the flexibility of copula functions to model the conditional dependence structure between the returns of Croatian and major European stock markets that the ordinary measures of (conditional) correlation are unable to measure. More specifically, we apply copula GARCH models where the return series are modeled as univariate GARCH processes and the dependence structure between the return series is defined by a copula function (see Patton, 2006a; Jondeau and Rockinger, 2006; Wu and Lin, 2010). We consider most common volatility models (GARCH, EGARCH, APGARCH, GJR-GARCH) to model the dynamic volatility structures of stock market returns. In order to determine the dependence structures, two commonly used copulas in economic and financial studies are estimated: the normal (or Gaussian) copula, that may be considered as the benchmark copula in the economic literature (see Patton, 2006a), and the "symmetrized Joe-Clayton" copula. There is abundant evidence in the literature (see Patton 2006a, Jondeau and Rockinger, 2006; Wu and Lin, 2010) that dependence between stock market returns is time-varying, we therefore estimate not only static copulas but also conditional (time-varying) normal and Symmetrized Joe-Clayton copulas.

An important benefit of using copulas to construct bivariate (multivariate) models is that the models used in the marginal distributions need not be of the same type

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<sup>2</sup> Pearson's correlation, as a measure of dependence, has several shortcomings (see Rachev et al., 2009; Necula, 2010). Firstly, this is only a linear measure of dependence. Secondly, independence between two random variables implies that the linear correlation is zero, while the converse is true only for a multivariate normal distribution. Thirdly, when the variance of returns tends to be infinite, that is, when extreme events are frequently observed, the linear correlation between these returns is undefined. Fourthly, linear correlation only measures the degree of dependence but does not clearly discover the structure of dependence.

as the model used for the copula. This is exploited by the semiparametric methods of copula estimation (developed by Genest et al. (1995) for i.i.d. data and Chen and Fan (2006a) for time series data), which also this paper applies. More specifically, the paper implements a semi-parametric method in which a non-parametric method is used to estimate the marginal distributions in the first stage (such as empirical distribution function), while a parametric method is used to estimate the copula parameters in the second stage.

The basic hypothesis of this paper is that dependence between the stock markets is dynamic and possibly non-linear and therefore cannot be properly captured by ordinary measures of dependence (the two frequently used in literature are the Pearson's correlation and dynamic conditional correlation modeled by multivariate GARCH). The main research questions we set are as follows: i) Is dependence between the Croatian and major European stock markets dynamic; ii) Does dependence in up (i.e. when returns across markets are simultaneously positive) and down markets (i.e. when returns across markets are simultaneously negative) differ?

Based on our evidence, dependence structure between Croatian and Major European stock markets is dynamic. We show that dependence in down markets usually exceeds dependence in up markets. We discuss implications of the findings for economic policy.

The paper consists of four related sections. After the introduction and literature review, the third section of the research provides the description of copula GARCH models and their benefits. The fourth section describes the data and the fifth analyses the empirical results of dependence modeling. The paper ends by outlining proposals and recommendations, as well as final reflections.

## **2. Literature review**

A theoretical foundation for the copula models is provided by Sklar's theorem (1959). The theorem states that for a given joint multivariate distribution function and the relevant marginal distributions, there exists a copula function that relates them (Patton, 2006a). The great attractiveness of the copula models thus comes from the flexibility of the copula models which permit marginal distributions that differ and allow the dependence structure to be flexible, i.e. it may be linear, nonlinear, or only tail dependent (Wu and Lin, 2010).

Patton (2006a,b) extended the standard concept of copulas to model dynamic dependence and proved the validity of Sklar's theorem for the conditional (time-varying) copula. In his time-varying copulas, the parameter(s) of a given copula

are specified analogous to a GARCH model. A similar modeling of dynamic dependence is suggested also by Jondeau and Rockinger (2006), while Rodriguez (2007), proposed a regime switching model for conditional copulas.

Due to its theoretical features the copula models have emerged as attractive models in finance for measuring various forms of dependence: i) between exchange rates (Lee and Long, 2006; Patton, 2006a; Boero et al., 2009), ii) contagion between financial markets (Durante and Jarowski, 2010), iii) risk management (Embrechts, et al., 2003; Rosenberg and Schuermann, 2006), iv) pricing of derivatives (Cherubini et al., 2004), and v) stock markets (Jondeau and Rockinger, 2006; Bartram et al., 2007; Necula, 2010). Among the few studies that investigate dependence between different stock markets, only some studied time evolution of dependence.

Jondeau and Rockinger (2006) applied a time-varying copula to the daily returns of four major stock markets, represented by indices S&P500, FTSE, DAX, and CAC. Their results suggest that conditional dependency depends on past realizations for European market pairs only. They also found that for the European stock markets dependence is higher and more persistent than for US stock market and more widely affected when returns move in the same direction than when they move in opposite directions.

Necula (2010) estimates the dependence structure between stock indices PX, SP500, BUX, and DAX statically and finds that the t-copula and the Gumbel-Clayton mixture copula are the most appropriate copula functions to capture the dependency structure of two financial return series.

Bartram et al. (2007) used a time-varying copula model to investigate the impact of the introduction of the Euro on the dependence between seventeen European stock markets during the period 1994-2003. They modeled marginal distribution by a GJR-GARCH-t and used the Gaussian copula for the joint distribution. Their results indicate that dependence across the market increased after the introduction of the common currency only for large equity markets, such as in France, Germany, Italy, the Netherlands and Spain. They also estimated time-varying dependence between the Euro and non-Euro European equity markets and found that the UK and Sweden, but not other countries outside the Euro area, exhibited an increase in equity market co-movement.

In this paper we follow Jandeau and Rockinger (2006) and Bartram et al. (2007) and thus apply not only static but also dynamic copula models to analyze dependence among investigated stock markets.

### 3. Methodology

#### 3.1. Definition of the copula and the conditional copula

The Sklar's theorem (1953) states, that for a given joint multivariate distribution function and the relevant marginal distributions, there exists a copula function that relates them (Patton, 2006a,b). To explain the Sklar's (1951) theorem, we will consider a bivariate setting. Let  $X$  and  $Y$  present two variables, and  $F_{XY}$  the joint distribution with margins  $F_X$  and  $F_Y$ , respectively. Then there exists a function  $C : [0,1]^2 \rightarrow [0,1]$  such that (Nelsen 2006, Patton 2006a)

$$F_{XY}(x,y) = C(F_X(x), F_Y(y)) \quad (1)$$

or

$$f_{xy}(x,y) = f_x(x)f_y(y) c(F_X(x) F_Y(y)), \quad (2)$$

where equation (1) decomposes a bivariate cumulative distribution function and equation (2) decomposes a bivariate density (where  $f_x(\cdot)$ ,  $f_y(\cdot)$ , and  $c(\cdot)$  are marginal and copula densities respectively).

It is also useful to think of copulas as joint distribution functions of standard uniform random variables  $U_1 = F(X_1)$  and  $U_2 = F(X_2)$ :

$$C(u,v) = Pr(U_1 \leq u, U_2 \leq v). \quad (3)$$

A copula is a function that satisfies the following conditions (see Necula, 2010):

- i)  $C : [0,1]^2 \rightarrow [0,1]$ , i.e. the outcome of uniform random variables falls into the interval  $[0,1]$ .
- ii)  $C(u,0) = C(0,v) = 0$ , i.e. one can also determine the value of a copula on the border of its domain. When one argument equals zero, the probability of any joint event must also be zero.
- iii)  $C(u,1) = u$  and  $C(1,v) = v$ ; i.e. when all but one of the inputs are equal to one, the joint probability must be equal to the marginal probability of the argument that does not equal one.
- iv) For any  $u_1, u_2, v_1, v_2 \in [0,1]$  with  $u_1 \leq u_2$  and  $v_1 \leq v_2$ , we have  $C(u_2,v_2) + C(u_1,v_1) - C(u_1,v_2) - C(u_2,v_1) \geq 0$ , i.e. the function must be increasing in all its arguments.
- v) For any point  $(u,v) \in [0,1] \times [0,1]$  the copula must lie in the interval  $C^-(u,v) \equiv \max(u + v - 1, 0) \leq C(u,v) \leq \min C(u,v) \equiv C^+(u,v)$ , where  $C^-$  and  $C^+$  are the minimum and maximum copula, respectively.

As copulas provide a convenient way to combine univariate marginals into a joint distribution, they are useful in constructing multivariate models of asset returns.

The Sklar's theorem can be extended for conditional distributions, thus allowing us to use copula theory in the analysis of time-varying conditional dependence (see Patton, 2006a,b). To explain conditional copula, let us introduce the conditioning variable  $W$ , which may be a vector. Let us denote the joint distribution of  $(X, Y, W)$  as  $F_{XYW}$ , the conditional distribution of  $(X, Y)$  given  $W$  as  $F_{XY|W}$ , and the conditional distributions of  $X|Y$  and  $Y|X$  as  $F_{X|W}$  and  $F_{Y|W}$ , respectively. Further let us assume that the dimension of the conditioning variable,  $W$ , is 1. The conditional bivariate distribution of  $(X, Y)|W$  can be derived from the unconditional joint distribution of  $(X, Y, W)$ :

$$F_{XY|W}(x, y|w) = f_w(w)^{-1} \frac{\partial F_{XYW}(x, y, w)}{\partial w}, \text{ for } w \in S, \tag{4}$$

where  $f_w$  is the unconditional density of  $W$  and  $S$  is the support of  $W$ .

The conditional copula of  $(X, Y)|W = w$ , where  $Y|W = w \sim F_{Y|W}(\cdot|w)$  and  $X|W = w \sim F_{X|W}(\cdot|w)$ , is the conditional joint distribution function of  $U \equiv F_{X|W}(X|w)$  and  $V \equiv F_{Y|W}(Y|w)$  given  $W = w^3$ . Patton (2006a) shows that a conditional copula has the properties of an unconditional copula, for each  $w \in S$ .

Patton (2006a) derives the extension of the Sklar's theorem for the conditional distributions. With definition of  $F_{X|W}(\cdot|w)$  and  $F_{Y|W}(\cdot|w)$  being the conditional distribution of  $X|W = w$  and  $Y|W = w$ , respectively,  $F_{XY|W}(\cdot|w)$  being the joint conditional distribution of  $(X, Y)|W = w$  and  $S$  the support of  $W$ , let us further assume that  $F_{X|W}(\cdot|w)$  and  $F_{Y|W}(\cdot|w)$  are continuous in  $x$  and  $y$  for all  $w \in S$ . Then there exists a unique conditional copula  $C(\cdot|w)$  such that

$$F_{XY|W}(x, y|w) = C(F_{X|W}(x|w), F_{Y|W}(y|w)|w), \forall (x, y) \in \bar{R} \times \bar{R} \text{ and each } w \in S \tag{5}$$

where  $\bar{R}$  is the extended real line  $\bar{R} \equiv R \cup \{\pm\infty\}$ .

### 3.2. The copula models

The main implication that is of great value in modeling the dependence between bivariate (multivariate) financial variables comes from the converse of the Sklar's theorem for conditional distributions. If we let  $F_{X|W}(\cdot|w)$  be the conditional distribution of  $X|W = w$ ,  $F_{Y|W}(\cdot|w)$  be the conditional distribution of  $Y|W = w$ , and  $\{C(\cdot|w)\}$  be a family of conditional copulas that is measurable in  $w$ , then the function  $F_{XY|W}(\cdot|w)$  defined by equation (5) is a conditional bivariate distribution function with conditional marginal distributions  $F_{X|W}(\cdot|w)$  and  $F_{Y|W}(\cdot|w)$ .

<sup>3</sup> The random variables  $U$  and  $V$  are the conditional "probability integral transforms" of  $X$  and  $Y$  given  $W$ , for which Fisher (1932) and Rosenblatt (1952) showed they have  $Unif(0,1)$  distribution regardless of the original distribution.

The copula function is employed to determine the dependence structures, and so its choice should rely on the characteristics of dependence observed in the data. Some commonly used copulas in economic and financial studies include elliptical copulas (such as Gaussian and Student-t copulas), Archimedean copulas (such as Gumbel copulas, Clayton and Frank copulas), and their combinations (for a review of different copulas see Nelsen (2006) or Schmidt (2007)). In this paper we will consider the normal (or Gaussian) copula that may be considered as the benchmark copula in the economic literature (Patton, 2006a; Wu and Lin, 2010), and the “symmetrized Joe-Clayton” copula. As there is abundant evidence in the literature (Forbes and Rigobon, 2002; Gilmore et al., 2008; Dajčman et al., 2012) that dependence between stock market returns is time-varying we are primarily interested in the conditional (time-varying) copulas. However, we will also estimate the static copulas.

The bivariate Gaussian (or normal) copula is defined as (Patton, 2006a):

$$C(u, v | \rho) = \Phi(\Phi^{-1}(u)\Phi^{-1}(v)) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)}\right\} dr ds, \quad (6)$$

$(-1 < \rho < 1),$

where  $u, v \sim Unif(0,1)$  and  $\Phi$  is the standard normal cumulative distribution function.

The normal copula is symmetric, has a linear measure of dependence (Pearson’s correlation coefficient,  $\rho$ ) and allows for equal degrees of positive and negative dependence, reflecting the asymptotic tail independence (Lee and Long, 2009). Patton (2006a) allows the copula parameters (and therefore dependence) to evolve over time via an evolution equation in which a constant, an autoregressive term, and a forcing variable is included:

$$\rho_t = \tilde{\Lambda}(\omega_\rho + \beta_\rho \rho_{t-1} + \alpha \frac{1}{10} \sum_{j=1}^{10} \Phi^{-1}(u_{t-j})\Phi^{-1}(v_{t-j})), \quad (7)$$

where  $\tilde{\Lambda}(x) \equiv (1 - e^{-x})(1 + e^{-x}) = \tanh(x/2)$  is the modified logistic transformation designed to keep the time varying correlation coefficient in  $(-1,1)$  at all times. The equation assumes that the copula parameter follows an ARMA (1,10)-type process: the term  $\rho_{t-1}$  is included to capture any persistence in the dependence parameter, while the mean of the product of the last 10 observations of the transformed variables  $\Phi^{-1}(u_{t-j})$  and  $\Phi^{-1}(v_{t-j})$  are used to capture any variation in dependence.

The next copula considered in this paper is the symmetrized Joe-Clayton copula (SJC) that was developed by Patton (2006a) as a modification of the “BB7” copula of Joe (1997). The Joe-Clayton copula is defined as

$$C_{JC}(u, v | \tau^U, \tau^L) = 1 - (1 - \{ [1 - (1 - u)^\kappa]^{-\gamma} + [1 - (1 - v)^\kappa]^{-\gamma} - 1 \}^{\frac{1}{\gamma}})^{\frac{1}{\kappa}}, \quad (8)$$

where  $\kappa = \frac{1}{\log_2(2 - \tau^U)}$ ,  $\gamma = -\frac{1}{\log_2(\tau^L)}$ , while  $\tau^U \in (0, 1)$  and  $\tau^L \in (0, 1)$  are parameters

that measure the upper and lower tail dependence<sup>4</sup>, i.e. the behavior of the random variables during extreme events. The modification of the above equation that yields the symmetric Joe-Clayton copula is defined as (Patton, 2006a)

$$C_{SJC}(u, v | \tau^U, \tau^L) = 0.5(C_{JC}(u, v | \tau^U, \tau^L) + C_{JC}(1 - u, 1 - v | \tau^U, \tau^L) + u + v - 1), \quad (9)$$

which nests symmetry ( $\tau^U = \tau^L$ ) as a special case, but also allows for asymmetric dependence, which may better fit the dependence structure in stock market returns as some empirical studies show (see Hong et al., 2007).

For the conditional dependence SJC copula, the following evolution equations are proposed following Patton (2006a)

$$\tau_t^U = \Lambda(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}|), \quad (10)$$

and

$$\tau_t^L = \Lambda(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}|), \quad (11)$$

where  $\tau_t^U$  and  $\tau_t^L$  are the time-varying upper tail and lower tail dependence measures, and  $\Lambda(x) \equiv (1 + e^{-x})^{-1}$  is the logistic transformation, used to keep  $\tau^U$  and  $\tau^L$  in  $(0, 1)$  at all times. Again, from the above equations follows that the upper and lower tail dependence parameters each follow something akin to a restricted ARMA(1, 10) process. The right-hand side of the model for the tail dependence evolution equation contains an autoregressive term,  $\beta_U \tau_{t-1}^U$  and  $\beta_L \tau_{t-1}^L$ , and a forcing variable, using the mean absolute difference between  $u_{t-j}$  and  $v_{t-j}$  over the previous 10 observations.

<sup>4</sup> Lower and upper tail dependence can be explained by the concept of quantile dependence (see Paton, 2008) that measures the probability of two variables both lying below or above a given quantile ( $q$ ) of their univariate distributions. It is defined as  $\tau(q) = C(q, q)/q$  for  $q \leq 0.5$  and  $\tau(q) = \{1 - 2q + C(q, q)\}/(1 - q)$  for  $q > 0.5$ . Lower and upper tail dependence can be defined as the limits of the quantile dependence measure ( $\lim_{q \rightarrow 0} \tau(q) = \tau^L$  and  $\lim_{q \rightarrow 1} \tau(q) = \tau^U$ ), if the limits exist.



### 3.3. A semiparametric method of estimation of the copula parameters

An important benefit of using copulas to construct bivariate (multivariate) models is that the models used in the marginal distributions need not be of the same type as the model used for the copula. This is exploited by the semiparametric methods of copula estimation (developed by Genest et al. (1995) for i.i.d. data and Chen and Fan (2006a,b) for time series data)<sup>5</sup>, which also this paper applies. More specifically, the paper implements a semi-parametric method in which a non-parametric method is used to estimate the marginal distributions in the first stage (such as empirical distribution function), while a parametric method is used to estimate the copula in the second stage. While copulas were originally developed for margins with i.i.d. observations, copulas have recently been applied also for time series following ARCH/GARCH-type processes (Jondeau and Rockinger, 2006; Wu and Lin, 2010). It has been shown that margins for i.i.d. can be replaced with those for ARMA-ARCH/GARCH filtered residuals (see Kim et al., 2007b, 2008; Boero et al., 2009).

In the first step of copula estimation we filter the stock market returns with a GARCH family model that best fits the volatility of the univariate return series<sup>6</sup>. We then estimate empirical distributions on the filtered returns, represented by the standardized residuals from these univariate models. Assuming that  $X$  and  $Y$  are i.i.d. variables with continuous marginal distribution functions, we estimate sample empirical distributions as (Genest et al., 1995; Patton, 2006a; Boero et al., 2009):

$$\hat{F}_X(x) = \frac{1}{T+1} \sum_{t=1}^T I_{\{X_t < x\}} \quad (12)$$

$$\hat{F}_Y(y) = \frac{1}{T+1} \sum_{t=1}^T I_{\{Y_t < y\}}, \quad (13)$$

<sup>5</sup> There is a variety of methods of copula estimation (for a review see Patton, 2009; Boero et al., 2009; Kim et al., 2007). Copula parameters can be estimated simultaneously by maximum likelihood or using a two-step maximum likelihood, in which the marginals are estimated in the first step and the dependence parameter is estimated in the second step using the copula. The latter method can further be classified into method where both steps are implemented parametrically, a method of non-parametrical copula estimation and a method of semi-parametrical copula estimation. Kim et al. (2004) show in a simulation study that the semiparametric method, which estimates the marginal distributions nonparametrically, is more robust than the method of simultaneous maximum likelihood estimation or the fully parametric two-step maximum likelihood estimation method.

<sup>6</sup> We chose among univariate GARCH family models that are usually used to fit the volatility of stock market returns: GARCH(1,1), GARCH(1,2), GARCH(2,1), GARCH(2,2), EGARCH (1,1), APGARCH(1,1) and GJR-GARCH(1,1). The best fitting model is then selected based on the Akaike information criteria. We do not present these models in detail, as this would exceed the dimensions of this paper. A concise presentation of all the models can be found in Bollerslev (2007).

where  $I_{\{\cdot\}}$  is an indicator function, and  $\hat{F}_X(x)$  and  $\hat{F}_Y(y)$  are pseudo-observations approximately distributed  $Unif(0,1)$ . Copula models are then estimated by maximum likelihood estimator

$$\hat{F}_{XY}(x,y; \theta) = C(\hat{F}_X(x), \hat{F}_Y(y); \theta), \tag{14}$$

where  $\theta$  represents a set of parameters measuring dependence for the bivariate set of pseudo- observations  $(\hat{F}_X(x), \hat{F}_Y(y))$ . The second step involves estimating  $\theta$  by maximizing the pseudo-log-likelihood function

$$\sum_{t=1}^T \log c(\hat{F}_X(x), \hat{F}_Y(y); \theta), \tag{15}$$

where  $c(\cdot, \theta)$  is the density of the copula (normal or SJC).

### 4. Data

The paper examines dependence between Eurozone countries, listed in Table 1, for the period from December 2, 2003 – January 27, 2012. We assume that the returns of the main national stock index represent the returns of the respective national stock market. The stock indices included are: ATX (Austria), CAC40 (France), DAX (Germany), FTSE100 (UK), FTSE-MIB (Italy), and CROBEX (Croatia). The stock market returns were calculated as the differences in the logarithms of the daily closing prices of indices  $(\ln(P_t) - \ln(P_{t-1}))$ , where  $P_t$  is an index value at time  $t$ . Days with no trading in any of the observed market were left out. The data for stock indices is Yahoo! Finance for all but CROBEX for which the source was the national stock exchange. Total number of data equals 1920.

Table 1: Descriptive statistics of stock indices returns

| Stock market index | Min     | Max    | Mean      | Std. deviation | Skewness | Kurtosis | Jarque-Bera statistics |
|--------------------|---------|--------|-----------|----------------|----------|----------|------------------------|
| ATX (Austria)      | -0.1637 | 0.1304 | 0.000179  | 0.01868        | -0.2891  | 11.4275  | 5,705.57***            |
| CAC40 (France)     | -0.0947 | 0.1059 | -0.000026 | 0.01577        | 0.1642   | 10.4452  | 4,440.75***            |
| CROBEX (Croatia)   | -0.1459 | 0.1478 | 0.000196  | 0.01505        | -0.3571  | 18.5568  | 19,391.80***           |
| DAX (Germany)      | -0.0743 | 0.1080 | 0.000278  | 0.01530        | 0.1162   | 9.4645   | 3,345.76***            |
| FTSE100 (U.K.)     | -0.0927 | 0.1079 | 0.000137  | 0.01349        | 0.1914   | 12.3669  | 7,027.12***            |
| FTSE-MIB (Italy)   | -0.0997 | 0.1087 | -0.000284 | 0.01618        | -0.1569  | 9.6681   | 3,563.07***            |

Notes: The Jarque-Bera statistics: \*\*\* indicate that the null hypothesis (of normal distribution) is rejected at a 1% significance level.

Source: Author’s calculations

All series display significant leptokurtic behavior as evidenced by the large kurtosis with respect to the Gaussian distribution. The Jarque-Bera test rejected the hypothesis of normally distributed time series. We also tested for stationarity of time series by the Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests and the results lead to conclusion of no unit root in returns series<sup>7</sup>.

The results of the Ljung-Box and the ARCH effect test show that the null hypothesis of no serial correlation in the squares of demeaned returns and no ARCH effects can be rejected thus indicating the (G)ARCH effects in the univariate volatility series.

Table 2: Ljung-Box and ARCH effect tests

| Stock market index | Ljung-Box Q <sup>2</sup> statistics (Q <sup>2</sup> (10)) | ARCH effect (5) |
|--------------------|---|-----------------|
| ATX                | 1,640.2***  | 442.2***        |
| CAC40              | 1,076.8***  | 322.9***        |
| CROBEX             | 1,147.5***  | 345.5***        |
| DAX                | 975.4***  | 304.6***        |
| FTSE100            | 1,343.2***  | 389.2*          |
| FTSE-MIB           | 1,181.4***  | 344.7***        |

Notes: The “portmanteau” test of Ljung and Box assesses the null hypothesis that a series of squares residuals exhibits no autocorrelation for 10 lags, against the alternative that some autocorrelation coefficient  $\rho(k)$ ,  $k = 1, \dots, 10$  is nonzero. \*\*\* indicate that the null hypothesis of no serial correlation can be rejected at a 1% significance level. Engle’s ARCH test reports the value of LM test statistics at 5 lags included: \*\*\* indicate that the null hypothesis of no ARCH effects can be rejected at a 1% significance level for up to the 5<sup>th</sup> lag.

Source: Author’s calculations

Next, we selected among the different GARCH family models that best fit the univariate volatility series. The best model is selected based on the Akaike information criteria.

<sup>7</sup> The results are not presented here, but can be obtained from the author. More properties of the returns of Croatian stock market can be found in Festić et al. (2012).

Table 3: Akaike information criteria for the different GARCH family models for the univariate stock market return volatility series

| Stock market index | GARCH (1,1) | GARCH (1,2) | GARCH (2,1)      | GARCH (2,2) | EGARCH (1,1) | APGARCH (1,1)    | GJR-GARCH (1,1) |
|--------------------|-------------|-------------|------------------|-------------|--------------|------------------|-----------------|
| ATX                | -10766.60   | -10764.60   | -10775.91        | -10773.91   | -10841.22    | -10790.53        | -10828.14       |
| CAC40              | -11308.71   | -11306.71   | -11311.60        | -11309.60   | -11452.47    | <i>-11453.75</i> | -11408.79       |
| CROBEX             | -11641,46   | -11653,64   | <i>-11639,46</i> | -11651,64   | -11616,80    | -11638,48        | -11638,49       |
| DAX                | -11302,78   | -11300,78   | <i>-11302,43</i> | -11300,48   | -11415,99    | -11427,69        | -11402,13       |
| FTSE100            | -12064.49   | -12062.49   | -12069.65        | -12061.51   | -12175.04    | <i>-12176.01</i> | -12153.72       |
| FTSE-MIB           | -11125,01   | -11125,01   | -11125,01        | -11428,54   | -11520,86    | <i>-11519,69</i> | -11513,30       |

Notes: The best fitting model, as indicated by AIC criteria, for each univariate return series is shaded.

Source: Author's calculations

As the results in Table 3 show, APGARCH(1,1) is the best fitting model of volatility of stock market returns of France, U.K., and Italy, GARCH(1,2) for Croatia and Germany, and EGARCH(1,1) for Austria.

## 5. Results and discussion

After obtaining standardized residuals of univariate return series, we estimated the parameters of bivariate copula models, the corresponding log-likelihood function (LLF), Akaike information criteria (AIC), and Bayesian information criteria (BIC). As the results indicate (Tables 4 through 6), conditional (time-varying) copulas are found to better fit the data as the constant copulas. The information criteria lead to the same conclusion regarding the best fit copula model for particular pair-wise stock markets.

Table 4: Results for the copula models – dependence between stock market returns of Croatia and Austria and Croatia and France

| Type of copula                       | Estimated parameters  | Stock market pair  |                    | Information criteria |         |         |              |         |         |
|--------------------------------------|-----------------------|--------------------|--------------------|----------------------|---------|---------|--------------|---------|---------|
|                                      |                       |                    |                    | CROBEX-ATX           |         |         | CROBEX-CAC40 |         |         |
|                                      |                       | CROBEX-ATX         | CROBEX-CAC40       | LLF                  | AIC     | BIC     | LLF          | AIC     | BIC     |
| Constant normal                      | $\bar{\rho}$          | 0.3248<br>(0.0212) | 0.2934<br>(0.0195) | -106.97              | -213.95 | -213.95 | -86.39       | -172.78 | -172.77 |
| Constant symmetrised Joe-Clayton     | $\bar{\tau}^U$        | 0.0863<br>(0.0280) | 0.0943<br>(0.0278) | -113.53              | -227.06 | -227.05 | -92.71       | -185.42 | -185.42 |
|                                      | $\bar{\tau}^L$        | 0.2004<br>(0.0245) | 0.1477<br>(0.0257) |                      |         |         |              |         |         |
| Time-varying normal                  | Constant              | -0.3112            | 0.1774             | -121.65              | -243.29 | -243.29 | -116.55      | -233.09 | -233.08 |
|                                      | $\alpha$              | 0.7142             | 0.4617             |                      |         |         |              |         |         |
|                                      | $\beta$               | -0.3418            | 1.0344             |                      |         |         |              |         |         |
| Time-varying symmetrized Joe-Clayton | Constant <sup>U</sup> | 0.6044             | 2.9827             | -135.55              | -271.09 | -271.07 | -109.49      | -218.98 | -218.96 |
|                                      | $\alpha^U$            | -13.8827           | -21.0586           |                      |         |         |              |         |         |
|                                      | $\beta^U$             | 2.4103             | -1.7443            |                      |         |         |              |         |         |
|                                      | Constant <sup>L</sup> | -2.0400            | -1.8422            |                      |         |         |              |         |         |
|                                      | $\alpha^L$            | -1.0387            | -1.8224            |                      |         |         |              |         |         |
|                                      | $\beta^L$             | 4.5413             | 4.3094             |                      |         |         |              |         |         |

Notes: In parentheses we report standard errors of the estimated parameters for the constant copula models calculated by a bootstrap method (with 100 repetitions) proposed by Chen and Fan (2006b), Rémillard (2010), and Patton (2012). There is no theoretical foundation for the calculation of the standard errors for the time-varying copula models (see Patton, 2012).

Source: Author's calculations

Table 5: Results for the copula models – dependence between stock market returns of Croatia and Germany and Croatia and U.K.

| Type of copula                       | Estimated parameters  | Stock market pair  |                     | Information criteria |         |         |              |         |         |
|--------------------------------------|-----------------------|--------------------|---------------------|----------------------|---------|---------|--------------|---------|---------|
|                                      |                       |                    |                     | CROBEX-ATX           |         |         | CROBEX-CAC40 |         |         |
|                                      |                       | CROBEX-DAX         | CROBEX-FTSE100      | LLF                  | AIC     | BIC     | LLF          | AIC     | BIC     |
| Constant normal                      | $\bar{\rho}$          | 0.2832<br>(0.0240) | 0.2796<br>(0.0228)* | -80.23               | -160.46 | -160.46 | -78.12       | 56.23   | -156.23 |
| Constant symmetrised Joe-Clayton     | $\bar{\tau}^U$        | 0.0933<br>(0.0313) | 0.0685<br>(0.0286)  | -89.86               | -179.71 | -179.70 | -91.47       | -182.94 | -182.93 |
|                                      | $\bar{\tau}^L$        | 0.1448<br>(0.0278) | 0.1678<br>(0.0236)* |                      |         |         |              |         |         |
| Time-varying normal                  | Constant              | 0.3374             | 0.0006              | -101.36              | -202.72 | -202.71 | -99.39       | -198.77 | -198.76 |
|                                      | $\alpha$              | 0.5990             | 0.0282              |                      |         |         |              |         |         |
|                                      | $\beta$               | 0.2643             | 2.0006              |                      |         |         |              |         |         |
| Time-varying symmetrized Joe-Clayton | Constant <sup>U</sup> | 1.2847             | 2.5534              | -99.28               | -198.55 | -198.53 | -105.74      | -211.48 | -211.46 |
|                                      | $\alpha^U$            | -13.1295           | -20.5121            |                      |         |         |              |         |         |
|                                      | $\beta^U$             | -1.2397            | -10.7000            |                      |         |         |              |         |         |
|                                      | Constant <sup>L</sup> | -1.4991            | -1.1240             |                      |         |         |              |         |         |
|                                      | $\alpha^L$            | -3.4252            | -6.0336             |                      |         |         |              |         |         |
|                                      | $\beta^L$             | 3.5609             | 3.5245              |                      |         |         |              |         |         |

Notes: See notes for Table 4.

Source: Author's calculations

We found that the time-varying normal copula yields the best fit for CROBEX-CAC40, CROBEX-DAX, and CROBEX-FTSE-MIB stock indices pairs, while the time-varying SJC copula is the best fit for the stock market pairs CROBEX-ATX, and CROBEX-FTSE100.

Table 6: Results for the copula models – dependence between stock market returns of Croatia and Italy

| Type of copula                       | Estimated parameters            |                            | Information criteria |         |         |
|--------------------------------------|---------------------------------|----------------------------|----------------------|---------|---------|
|                                      |                                 |                            | LLF                  | AIC     | BIC     |
| Constant normal                      | $\bar{\rho}$                    | 0.2825<br>(0.0225)         | -79.78               | -159.56 | -159.56 |
| Constant symmetrised Joe-Clayton     | $\bar{\tau}^U$                  | 0.0894<br>(0.0308)         | -86.90               | -173.79 | -173.78 |
|                                      | $\bar{\tau}^L$                  | 0.1417<br>(0.0266)         |                      |         |         |
| Time-varying normal                  | Constant<br>$\alpha$<br>$\beta$ | 0.0074<br>0.1419<br>1.9431 | -118.10              | -236.19 | -236.18 |
| Time-varying symmetrized Joe-Clayton | Constant <sup>U</sup>           | 0.2028                     | -105.32              | -210.63 | -210.61 |
|                                      | $\alpha^U$                      | -10.4770                   |                      |         |         |
|                                      | $\beta^U$                       | 1.6746                     |                      |         |         |
|                                      | Constant <sup>L</sup>           | -0.9257                    |                      |         |         |
|                                      | $\alpha^L$                      | -5.2353                    |                      |         |         |
|                                      | $\beta^L$                       | 3.0478                     |                      |         |         |

Notes: See notes for Table 4.

Source: Author's calculations

We plot (Figures 1 through 5 – see Appendices) the estimated conditional normal copula dependence measure (correlation) and the conditional upper and lower tail dependencies from the time-varying SJC copula model. We notice a substantial time variation in correlation as well as tail dependencies, which supports the findings in the existent literature that the dependence between stock markets is time-varying (e.g. Wu and Lin, 2010; Patton, 2012). In the investigated time period the Croatian stock market was on average most correlated with the Austrian and French stock markets (see Table 7). The lower tail dependence was on average higher than the upper tail dependence for the stock market return pairs CROBEX-ATX, CROBEX-CAC40, and CROBEX-FTSE-MIB, while the opposite is true for the CROBEX-DAX, and CROBEX-FTSE100 return pairs (Table 8). For the three former stock indices pairs we also note that the tail dependence measure is more volatile as measured by standard deviation.

The lower tail dependence ranges from zero to around 0.72, and the upper tail dependence from zero to about 0.77 for stock market pairs within the sample period.

This indicates that the probability of these joint extreme negative movements in the stock markets can range from near zero to about 72 percent, while extreme positive from zero to 77 percent.

Table 7: Descriptive statistics of the time-varying correlation variable

| Stock market pair | Min     | Max    | Mean   | Std. dev. |
|-------------------|---------|--------|--------|-----------|
| CROBEX-ATX        | -0.1172 | 0.7642 | 0.3156 | 0.1513    |
| CROBEX.CAC40      | -0.1209 | 0.7335 | 0.2859 | 0.1518    |
| CROBEX-DAX        | -0.1967 | 0.7346 | 0.2750 | 0.1290    |
| CROBEX-FTSE100    | -0.1012 | 0.7047 | 0.2734 | 0.1295    |
| CROBEX-FTSEMIB    | -0.2974 | 0.6506 | 0.2642 | 0.1807    |

Source: Author's calculations

Table 8: Descriptive statistics of the tail dependence measure variables

| Stock market pair | Lower tail dependence ( $\tau_l^L$ ) |        |        |           | Upper tail dependence ( $\tau_u^U$ ) |        |        |           |
|-------------------|--------------------------------------|--------|--------|-----------|--------------------------------------|--------|--------|-----------|
|                   | Min                                  | Max    | Mean   | Std. dev. | Min                                  | Max    | Mean   | Std. dev. |
| CROBEX-ATX        | 0.1121                               | 0.5773 | 0.2139 | 0.0715    | 0.0018                               | 0.7651 | 0.1023 | 0.1299    |
| CROEX-CAC40       | 0.0885                               | 0.6876 | 0.1896 | 0.0905    | 0.0015                               | 0.6364 | 0.0976 | 0.1101    |
| CROBEX-DAX        | 0.0010                               | 0.1448 | 0.0011 | 0.0033    | 0.001                                | 0.1543 | 0.0777 | 0.0767    |
| CROBEX-FTSE100    | 0.001                                | 0.1678 | 0.0011 | 0.0038    | 0.001                                | 0.1308 | 0.0659 | 0.0649    |
| CROBEX-FTSEMIB    | 0.0339                               | 0.7198 | 0.1439 | 0.0891    | 0.008                                | 0.6955 | 0.1019 | 0.0919    |

Source: Author's calculations

Notably, the lower and upper tail dependencies paths are similar in patterns, differing only in scale (see Figures 1 through 5 in Appendices). While the mean lower tail dependence between Croatian and other investigated European stock markets is on average higher than upper tail dependence, a formal test of asymmetric dependence rejects the hypothesis that the difference in tail dependencies are different<sup>8</sup>.

Figures 1 through 5 (see Appendices) can be used to investigate whether extreme financial market events had an impact on the correlation and tail dependencies between the Croatian and the European stock markets. For this purpose we drew

<sup>8</sup> The test results are not presented here, but can be obtained from the author. The test is based on Patton (2012).

a vertical line in the Figures 1 through 5 (see Appendices), indicating the start of the most severe financial market turmoil events in the European financial markets. Evidently, the outbreak of the global financial crisis (the collapse of Lehman Brothers is taken as the trigger event) coincides with a major increase in dependence measures for all stock market pairs that remained at higher levels all until second half of 2009. Similarly, also other indicated financial market turmoils (the outbreak of Greece's and Italy's debt crisis) coincided with an increase in correlation and tail dependencies across the stock markets.

The results of this paper confirm the findings of other studies that dependencies across stock markets are time-varying and that extreme financial market events increase dependence between the markets (Ang and Chen, 2002; Bae et al., 2003; Corsetti et al., 2005; Bekaert et al., 2005; Jondeau and Rockinger, 2006; Dungey and Martin, 2007; Hong et al., 2007; Dajcman et al., 2012).

These findings are relevant for individual and institutional financial market investors present in the investigated stock markets, for the purpose of portfolio management, and for the supervisory authorities and central banks, because of implications the dependence between the financial markets and the institutions of systemic importance have on the stability of financial markets. Finding larger dependence between stock markets in case of negative shocks in the markets indicated that spillovers of negative spillovers are more likely than spillovers of positive shocks. This calls for a greater role of policies that try to stabilize financial markets in the shortest possible time after the shock, a greater emphasis being put on macroprudential policies, but also for a great international coordination of economic policies.

## **6. Conclusion**

We found a substantial time variation in correlation as well as tail dependencies, which supports the findings in the existent literature that the dependence between stock markets is time-varying and may be different in the market turmoil periods. The basic hypothesis of this paper, that dependence between the stock markets is dynamic and possibly also non-linear and therefore cannot be properly captured by ordinary measures of dependence, is therewith proved. The results also indicate that during the turbulent times, when returns on stock markets either fall or rise extremely, the dependence between stock markets significantly increases. The lower and upper tail dependencies paths are similar in patterns, differing only in scale. Our study complements the existent empirical literature on the dynamic comovement of the Croatian stock market with the European stock markets with a novel copula GARCH approach. We show that univariate stock market returns volatilities may have a different best-fit ARCH/GARCH model and therefore the



copulas provide a convenient way to combine univariate marginals into a joint distribution.

The study has some limitations. Namely, due to space limitation we applied only four copulas, most frequently used to models asset returns. There, however, are other interesting copulas, for instance Student's copula that might successfully model the dependence between stock market returns. The future research could test the success of other copulas in this purpose. Secondly, the results are static in sense that they test for long memory in stock (indices) returns for the whole observed period only. The fractal structure of the time series may, however, change through time. The future research should therefore develop and apply also the dynamic versions of the long memory tests.

The findings of this paper are relevant for individual and institutional financial market investors present in Croatian and other investigated European stock markets as incorporating the asymmetry of correlations into portfolio decisions can add substantial economic value to investors. The dependence structure is also important for supervisory authorities and central bank, because of implications the dependence between the financial markets and the institutions of systemic importance have on the stability of financial markets.

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## Međusobna ovisnost hrvatskog i pojedinih europskih dioničkih tržišta – Kopula GARCH pristup

Silvo Dajčman<sup>1</sup>

### Sažetak

*Cilj ovog rada je analizirati strukturu međusobne ovisnosti prinosa hrvatskog i pet europskih dioničkih tržišta (austrijskog, francuskog, njemačkog, talijanskog i britanskog). Ishodišna hipoteza jest, da je međusobna ovisnost dinamična i vjerojatno nelinearna i stoga ne može biti korektno ocjenjena primjenom običnih mjera međuzavisnosti, kao što su Pearsonova korelacija i dinamična korelacija. Umjesto toga, u ovom se radu primjenjuje pristup kopula GARCH, s univarijantnim GARCH modeliranjem prinosa pojedinih tržišta, a struktura međusobne ovisnosti modelira se kopula funkcijama. Upotrijebljene su četiri različite kopula funkcije – konstantna i kondicionalna normalna i simetrična Joe-Claytonova (SJC) kopula – koje se ocjenjuju semi-parametričnom metodom. Rezultati studije pokazuju, da najbolju ocjenu međusobne ovisnosti između indeksa CROBEX-CAC40, CROBEX-DAX i CROBEX-FTSEMIB pruža dinamična normalna kopula, a između CROBEX-ATX i CROBEX-FTSE100 dinamična SJC kopula. Jedan od rezultata ove studije ukazuje na to da vjerojatnost simultanog ekstremnog pozitivnog i ekstremno negativnog prinosa na hrvatskom i jednom od drugih istraženih europskih dioničkih tržišta može porasti na 77 % u trenutku ekstremne volatilnosti na dioničkom tržištu.*

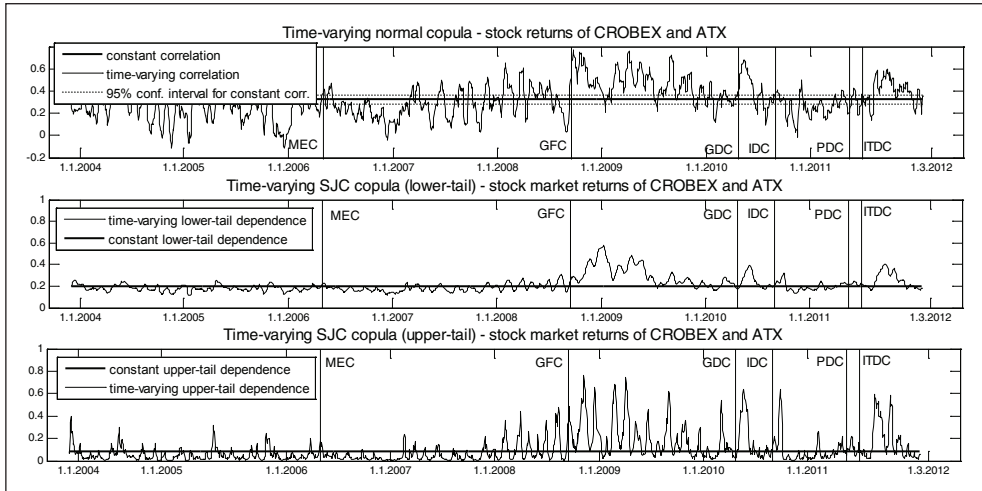
**Ključne riječi:** dioničko tržište, međusobna ovisnost, kopula GARCH

**JEL klasifikacija:** G11, G15

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## Appendices

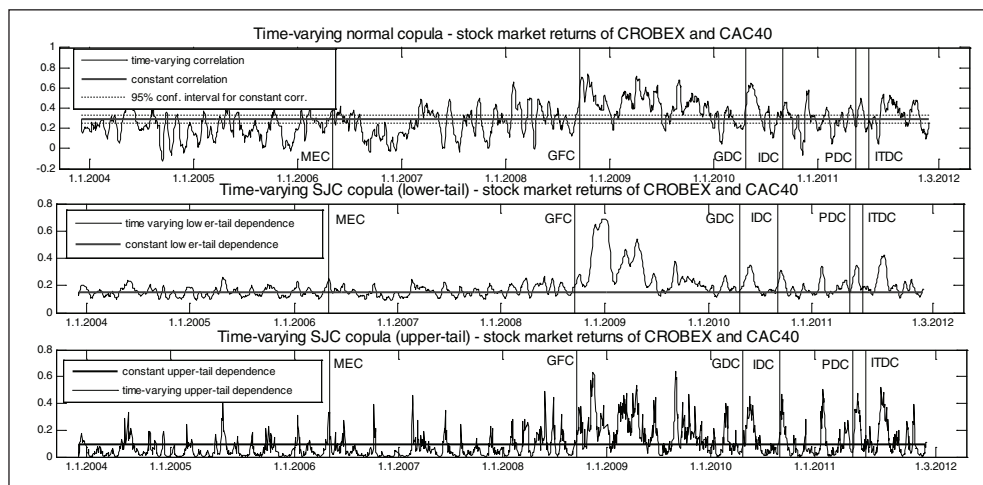
Figure 1: Time-varying dependence (correlation and tail dependence) between Croatian and Austrian stock market



Notes: On the time axis, the following financial turmoil events are denoted: MEC = the Middle East financial markets crash (the start of May 2006 is denoted), GFC = the global financial crisis (September 16, 2008, the collapse of Lehman Brothers, is denoted), GDC = the Greek debt crisis (April 23, 2010, when the Greek government requested a bailout from the EU/IMF is denoted), IDC = Ireland's debt crisis (September 1, 2010, when the Irish government started negotiations for a bailout with the ECB/IMF is denoted), PDC = Portugal's debt crisis (May 16, 2011 is denoted, when Eurozone leaders approved a bailout of financial help for Portugal is denoted), ITDC = Italy's debt crisis (the start of July 2011 is denoted, when financial market expectations for an Italian bailout request were mounting and the bond yields reached a level, at which other countries with sovereign debt crisis had requested help). We draw a 95% confidence interval in the first subplot (time-varying correlation) that the correlation is equal to the constant correlation estimate (correlation estimate from the constant normal copula).

Source: Author's calculations

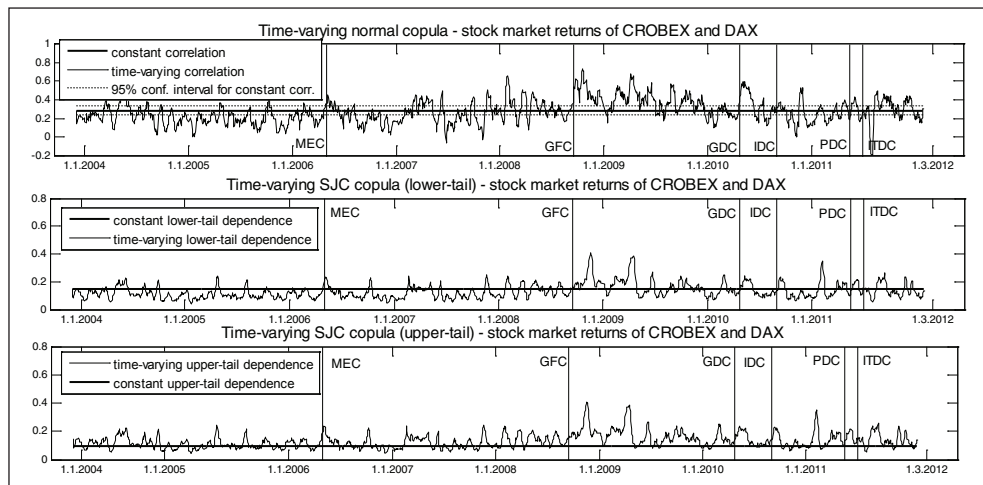
Figure 2: Time-varying dependence (correlation and tail dependence) between Croatian and France's stock market



Notes: See notes for Figure 1.

Source: Author's calculations

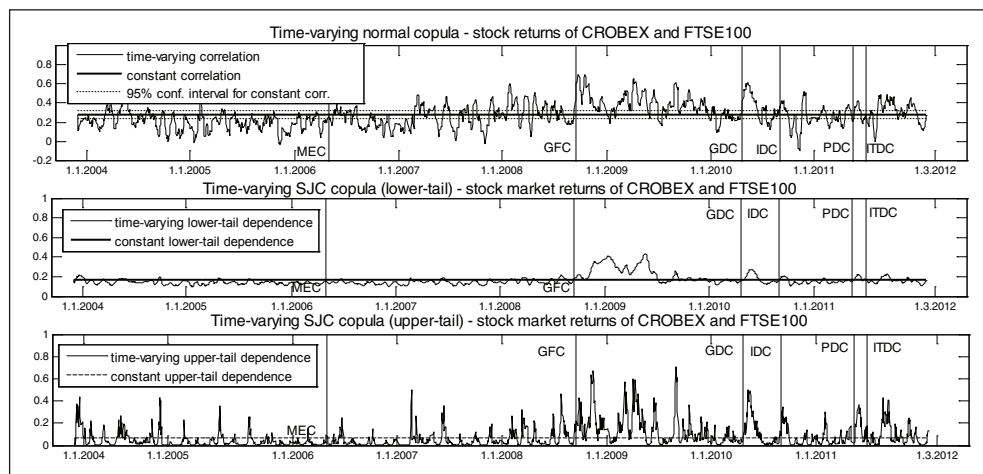
Figure 3: Time-varying dependence (correlation and tail dependence) between Croatian and German stock market



Notes: See notes for Figure 1.

Source: Author's calculations

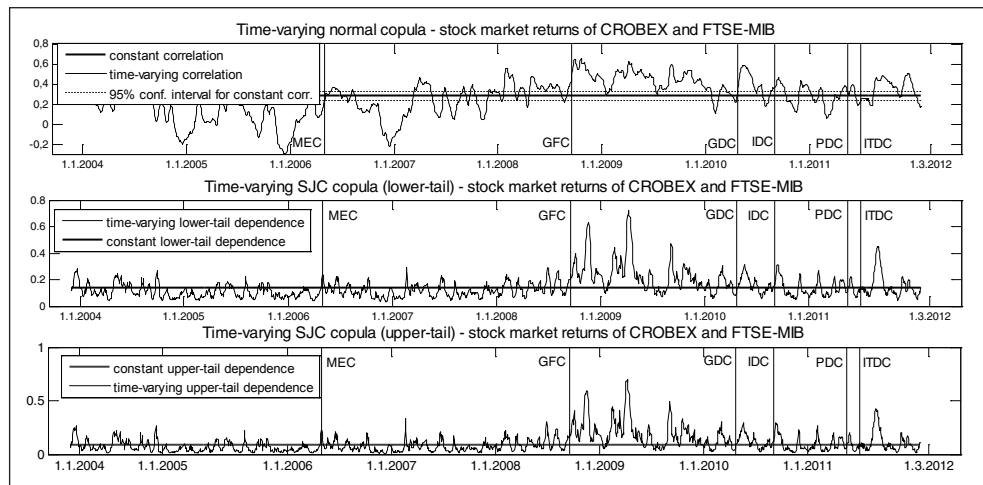
Figure 4: Time-varying dependence (correlation and tail dependence) between Croatian and U.K. 's stock market



Notes: See notes for Figure 1.

Source: Author's calculations

Figure 5: Time-varying dependence (correlation and tail dependence) between Croatian and Italy's stock market



Notes: See notes for Figure 1.

Source: Author's calculations